Secular Economic Changes and Bond Yields

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Abstract

We build a model of bond yields in an economy with secular changes to inflation, real rate and output growth. Long-run restrictions identify nominal shocks that do not influence the long-run real rate and output growth. Before the anchoring of inflation around the mid-1990s, nominal shocks lifted the output gap and inflation. This led to a higher and steeper yield curve because the short rate was expected to peak after several quarters, following declines in the responses of growth and inflation. With inflation anchored, nominal shocks have small impacts on inflation, output and bond yields, mostly via the term premium.

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1 Introduction

What are the economic forces behind the secular decline in bond yields? In the early 1980s, bond yields exhibited cyclical changes around declines in the long-run level of inflation π_t^*. This level stabilized at around 2 percent during the 1990s. More recently, bond yields exhibited cyclical changes around declines in the long-run level of the real rate r_t^*. Cieslak and Povala (2015) and Bauer and Rudebusch (2020) show that it is essential to disentangle these cyclical and secular variations to understand what drives bond yields. Intuitively, ignoring the secular variations in π_t^* and r_t^* means that the continued decline in yields since the early 1980s is largely attributed to a decline in the term premium.

We contribute to better understanding the secular decline in bond yields. We implement long-run restrictions pioneered by Blanchard and Quah (1989) to identify nominal and real shocks in a model of bond yields that has, in the spirit of Bauer and Rudebusch (2020), both cyclical and secular components. Our approach is distinct because (i) we connect bond yields to macroeconomic variations, (ii) we let the cyclical and secular components share the same nominal and real shocks that we identify from the data and (iii) the shocks have loadings that vary over time. Hence, both the real and nominal shocks drive cyclical variation as well as the secular decline in bond yields via their impacts on π_t^* and r_t^*. In the results, when we let the loadings of these shocks vary over time, we find that nominal shocks play a large role in driving the economy and bond yields early in our sample but that these impacts essentially disappear later in our sample, once inflation anchored.

To understand these results, which we explain below, it is useful to first see the four building blocks in our approach. First, we use a small-scale representation of the economy that is based on the short rate, inflation and output, where each variable is the sum of one cyclical and one secular component, which are both unobserved.[1] The cyclical components

[1]The presence of secular variations in output is also well known and has been extensively studied since Nelson and Plosser (1982). See also the discussion and the references

are correlated and related to each other. The secular components are also correlated. Like in Jordà and Taylor 2019, using this representation of the economy connects bond yields to underlying cyclical and secular macroeconomic changes based on a common set of assumptions.

Second, and in contrast with Jordà and Taylor 2019, we let the cyclical and secular components share the same macroeconomic shocks. This commonality arises in equilibrium models that exhibit delays in the adjustments to permanent shocks. Our identification strategy is close in spirit to the long-run restrictions pioneered by Blanchard and Quah (1989). We implement the definition that nominal shocks are neutral: these shocks can influence cyclical fluctuations in the output, inflation and interest rates as well as secular fluctuations in inflation, but they leave the long-run levels of output growth and real rate unchanged.

Third, we let the loadings of nominal and real shocks on the secular and cyclical components change over time in the way that is similar to Primiceri (2005), but in a different context. Time-varying loadings means that the impact of the shocks on the economy can change over time. In addition, having time-varying loadings implies that the volatilities of the short rate, inflation and output change over time, even if the shocks have constant variances. Indeed, Stock and Watson (2007) and Wright (2011) emphasize time-varying loadings in univariate reduced-form models of inflation.

Fourth, to derive bond prices, we assume that the nominal and real shocks are the sources of risk and that their risk prices are functions of the macro variables. Bond yields satisfy no-arbitrage and are functions of secular and cyclical components. However, we impose the restriction that the expected bond returns are stationary: they are not functions of secular components. Hence, consistent with the evidence by Cieslak and Povala (2015) and Bauer and Rudebusch (2020), our model implies that π_t^* and r_t^* enter in bond yields but not the term premium. This restriction makes the model parsimonious, improves

in Cochrane (1988) and Morley, Nelson and Zivot (2003).

the accuracy of parameter estimates and also prevents the variance of the expected bond returns from diverging with the investment horizon.[2] Finally, we follow earlier work and introduce in this representation an exogenous factor to capture the variations in bond yields that are uncorrelated with the macroeconomic shocks. We label this factor a "term structure factor" because, by design, it only influences the term premium and not the expectation component of bond yields. This prevents this exogenous shock from entering the dynamics of the short rate, output and inflation. In the results, this term structure factor explains a large share of the variations in bond yields at the quarterly horizon, but this share declines at longer horizons, which is consistent with the seminal macro-finance results of Ang and Piazzesi (2003).

Empirically, we find that the impacts of nominal shocks on bond yields are very different after inflation anchored in the 1990s. We quantify three reasons that operated essentially simultaneously to explain this change. The first channel between nominal shocks and bond yields acts on long-term inflation. As expected, π_t^* estimates decline over time, starting in our sample at around 6 percent and reaching a bottom close to 2 percent in the mid-1990s. Nominal shocks explain around two thirds of this decline. The reason behind π_t^*'s bottoming is that the sensitivity of long-run inflation to nominal shock gradually declines and essentially disappears by the 2000s. We pick two dates to illustrate this change: 1984Q1 and 2007Q1. In 1984, nominal shocks have a permanent impact on inflation. A one-standard deviation nominal shock lifts π_t^* by around 0.1 percentage points. To put this number in context, the impact on actual inflation peaks above 0.3 percentage points. Move forward to 2007: the response of π_t^* to nominal shocks becomes close to zero and insignificant.

The second channel between nominal shocks and bond yields acts on the cyclical responses of the economy. Early in our sample, there is a strong cyclical response. A one-

[2] We impose that the volatility of interest rates converges with the forecast horizon, which is not the case in existing models of bond prices with secular changes.

standard-deviation nominal shock lifts inflation but also the output gap by 0.5 percentage points on impact; the effect peaks at around 0.7 and lasts a few years. In response, the short rate increases by around 0.45 percentage points, peaks around 0.7 and the effect lasts a few years. By contrast, the same nominal shock only causes small, transitory and statistically insignificant responses in 2007. When we decompose the innovations to the cyclical component of the short rate date by date, we find that nominal shocks played a dominant role until the mid-1990s but then become much less important later on. Overall, these changes are consistent with the theoretical argument in Clarida, Gali and Gertler (2000) that the post-1979 policy responses to inflation stabilized output and inflation. These changes also reflect the empirical results in Cogley and Sargent (2005) showing that policy activism by the Federal Reserve in that period contributed to a fall in the level and the persistence of inflation.

Combined, these channels imply that the response of bond yields to nominal shocks changes dramatically. Early in our sample, this response leads to a higher level of the yield curve and a steeper slope between the short rate and longer-term yields. However, for the 2-year and 10-year bonds, the initial impact on the expectation component is around 0.6 and 0.4 percentage points, respectively. The difference between these two responses is due to the response of expectations about future short rates: it increases the curvature and causes an inversion of the yield curve between 2- and 10-year maturities. This inversion implies that the short rate will continue to rise but that the growth and inflation rates will decrease. All of these effects of nominal shocks become much smaller and statistically insignificant later in the sample. Note that the observation that the expectation component drives the predictive content of a yield curve inversion for future growth is consistent with the results by Ang, Piazzesi and Wei (2006). Also, the lack of inversion of the yield curve after a nominal shock later in our sample is consistent with the suggestion by Haubrich (2006) that the predictive content of a yield curve inversion has become weaker due to the rising Federal Reserve credibility in taming inflation.

The third channel acts on the term premium. For the 2- and 10-year bonds, the impact early in the sample is around 0.1 and 0.2 percentage points initially and peaks close to 0.25 and 0.4, respectively. This steepens the yield curve further but mutes its inversion. Overall, accounting for the expectation and term premium components, the responses of bond yields to nominal shocks considerably weaken after the mid-1990s. The initial all-in impact is almost halved on the 2-year yield, cut by one third on the 10-year yield and much less persistent in both cases.

Given that the secular inflation π_t^* had stabilized, the continued decline of bond yields during the 2000s is attributed to the decline in r_t^*, a point made clear by Bauer and Rudebusch (2020). We find that changes to r_t^* in the first half or so of our sample are driven by real shocks uncorrelated with changes to the long-run growth rate. However, the changes to r_t^* in the second half of the sample appear largely driven by shocks correlated with changes to potential output, suggesting that the role of technology, capital accumulation and demographics played a larger role during that period.

The rest of this paper is organized as follows. Section 2 introduces our small-scale model of the US economy with secular and cyclical components and details our specification of the bond prices. Section 3 introduces the data, reports the secular and cyclical components we recover, discusses the role of the different shocks and analyzes the response of the bond yields to these shocks. The online appendix contains many derivations and additional details about the estimation. Section 4 concludes.

Literature

The earlier works of Kozicki and Tinsley (2001) and Kozicki and Tinsley (2006) introduce a shifting endpoint to capture secular changes in the short rate and in the inflation dynamics, respectively. We use their definition of a shifting endpoint, which is based on the Beveridge-Nelson decomposition, to define the secular components in our small-scale economy. Laubach and Williams (2003) estimate a shifting endpoint for the real rate, and the evidence is expanded to other advanced economies by Holston, Laubach and Williams (2017) as well as Negro, Giannone, Giannoni and Tambalotti (2019), documenting a common decline across advanced economies. This follows early evidence in Garcia and Perron (1996) of distinct regimes in the behavior of the US real interest rate. Our π_t^* and r_t^* estimates are similar to existing results. Jordà and Taylor (2019) investigate a small-scale representation of the economy with distinct endpoints for output, inflation and the real rate but do not attempt to identify structural shocks.

van Dijk, Koopman, van der Wel and Wright (2014) allow for a shifting endpoint in a model of bond prices to improve interest rate forecasts. Cieslak and Povala (2015) provide compelling evidence that controlling for secular changes in expected inflations, based on either a discounted moving average or long-horizon survey forecasts, provides a more accurate measure of the bond risk premium. Christensen and Rudebusch (2019) recover r_t^* estimates using a DTSM for prices of individual inflation-indexed bonds. Joyce, Kaminska and Lildholdt (2012) provide an early application to UK real bonds. Bauer and Rudebusch (2020) provide evidence that secular real interest rate variations are responsible for the very high persistence of interest rates, and they offer a parsimonious dynamic term structure model (DTSM) of the yield curve using three standard yield factors and one common stochastic trend. Adding to this work, we explore the macroeconomic determinants of secular changes in bond yields.

Primiceri (2005) and Cogley and Sargent (2005) use a vector auto-regression with time-varying parameters to study the interaction between the US economy and monetary policy. Primiceri (2005) finds that the systematic responses of the short rate to inflation and unemployment trend toward a more aggressive behavior over time. Cogley and Sargent (2005) document changes over time in the natural rate of unemployment, a core rate of inflation, the persistence in inflation and the degree monetary policy activism. This work focuses on the periods before and after the Federal Reserve chairmanship of Paul Volcker. We focus on the responses of bond yields in the period starting with the chairmanship of

Alan Greenspan, when the Federal Reserve maintained its degree of activism.

2 Model

We develop a small-scale representation of an economy where the short rate, the inflation rate and output have joint dynamics. Each variable has a stationary and a non-stationary component, which we label "cyclical" and "secular", respectively. One reason for modeling these components together is to recover internally consistent estimates of the secular changes in the nominal rate, the inflation rate and the real rate. Another reason is that the results avoid the criticism in Coibion, Gorodnichenko and Ulate (2018) that existing estimates of secular component exhibit predictable transitory responses to demand shocks.

In the following, we first introduce a restricted version of the model that has constant loadings. Then, in a second step we introduce the time-varying loadings. Considering the restricted case first simplifies some of the exposition. It is useful to estimate both cases to understand the role of the time-varying loadings.

2.1 Notations

We label the short rate i_t, the inflation rate π_t and output y_t. Each of the macroeconomic variable has a Beveridge-Nelson decomposition, given by:

$$\begin{aligned} i_t &= i_t^* + \tilde{i}_t \\ \pi_t &= \pi_t^* + \tilde{\pi}_t \\ y_t &= y_t^* + \tilde{y}_t. \end{aligned} \tag{1}$$

Equation 1 is a statistical representation of the observed macro variables in terms of unobserved components. These components have no economic content yet. The following sections construct a system of dynamic equations where the variables with the * symbol represent the secular components and the variables with the ˜ symbol represent the cyclical components. This joint system provides the economic content and allows us to identify the unobserved components from the data. One feature of this system is that the components share the same set of shocks and can be correlated with each other. This contrasts with the assumption in most existing unobserved component models that innovations to the secular and cyclical components are uncorrelated.

2.2 Conditional Means

Stack the observed variables i_t, π_t and y_t in the vector M_t. This section specifies the joint conditional mean dynamics:

$$M_t \equiv E_{t-1}[M_t] + \Sigma \varepsilon_t, \tag{2}$$

with shocks $\varepsilon_t \sim N(0, I)$ where I is the identity matrix. In a first step, we specify the joint conditional mean of the secular components. In the second step, we specify the joint conditional mean of the cyclical components.

Secular Components Define the expected growth rate $g_t^* \equiv E_t[y_{t+1}^* - y_t^*]$ and stack the variables i_t^*, π_t^* and g_t^* in the vector $\bar{M}_t$, for which we assume the following dynamics,

$$\bar{M}_t \equiv \begin{bmatrix} i_t^* \\ \pi_t^* \\ g_t^* \end{bmatrix} = \begin{bmatrix} i_{t-1}^* \\ \pi_{t-1}^* \\ g_{t-1}^* \end{bmatrix} + \begin{bmatrix} \sigma'_{i^*} \\ \sigma'_{\pi^*} \\ \sigma'_{g^*} \end{bmatrix} \varepsilon_t = \bar{M}_{t-1} + \bar{\Sigma} \varepsilon_t. \tag{3}$$

Following Laubach and Williams (2003) among others, we take that the growth of potential output Δy^* is integrated of order 1:

$$\Delta y_t^* = g_{t-1}^* + \sigma'_{y^*} \varepsilon_t. \tag{4}$$

This nests the case where the level of potential output y_t^* is integrated of order 1 if g_t^* is constant. Equation 3 means that each element of $\bar{M}_t$ matches the definition of a shifting endpoint in the sense of Kozicki and Tinsley (2001). That is, for any element x_t^* of $\bar{M}_t$, we have that

$$x_t^* = \lim_{h \to \infty} E_t[x_{t+h}], \tag{5}$$

and that g_t^* is the shifting endpoint for output growth. Also, because we model these variables together, we can recover internally consistent estimates of the secular component of the real rate $r_t^* = i_t^* - \pi_t^*$.

Cyclical Components We stack the cyclical components $\tilde{i}_t$, $\tilde{\pi}_t$ and $\tilde{y}_t$ in the vector $\tilde{M}_t$, which we assume has standard stationary VAR dynamics, given by:

$$\tilde{M}_t \equiv \begin{bmatrix} \tilde{i}_t \\ \tilde{\pi}_t \\ \tilde{y}_t \end{bmatrix} = \Phi(L) \begin{bmatrix} \tilde{i}_t \\ \tilde{\pi}_t \\ \tilde{y}_t \end{bmatrix} + \begin{bmatrix} \sigma'_{\tilde{i}} \\ \sigma'_{\tilde{\pi}} \\ \sigma'_{\tilde{y}} \end{bmatrix} \varepsilon_t = \Phi(L)\tilde{M}_t + \tilde{\Sigma}\varepsilon_t, \tag{6}$$

where $\Phi(\cdot)$ is a polynomial function, L is the lag operator and the unconditional mean of $\tilde{M}_t$ is zero. The shocks $\varepsilon_t \sim N(0, I)$ are the same as in Equation 3.

For comparability, we embed the cross-equation restrictions used by Laubach and Williams (2003). First, the reduced-form specification for inflation is given by:

$$E_{t-1}\left[\tilde{\pi}_t\right] = b_1\tilde{\pi}_{t-1} + b_2\tilde{\pi}_{t-2,4} + b_y\tilde{y}_{t-1}, \tag{7}$$

where $\tilde{\pi}_{t-2,4}$ is the inflation rate computed between $t-2$ and $t-4$. Equation 7 ties inflation to the lags of inflation and output gap. Second, the reduced-form specification of the output gap is given by:

$$E_{t-1}\left[\tilde{y}_t\right] = a_1\tilde{y}_{t-1} + a_2\tilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}\tilde{r}_{t-j}, \tag{8}$$

where $\tilde{r}_t$ is the cyclical component of the real rate $\tilde{r}_t = r_t - r_t^*$, which can be computed by using $\tilde{r}_t = \tilde{i}_t - E_t[\tilde{\pi}_{t+1}]$. Hence, Equation 8 ties the output gap to lags of the output gap and monetary stance $\tilde{r}_t$. Third, we add to the framework of Laubach and Williams (2003) a reduced-form forward-looking specification of the short-rate gap in the spirit of the Taylor rule (Clarida, Galí and Gertler, 1998), given by:

$$E_{t-1}\left[\tilde{i}_t\right] = \delta_\pi E_{t-1}\left[\tilde{\pi}_t\right] + \delta_y E_{t-1}\left[\tilde{y}_t\right] + \phi_z\left(\tilde{i}_{t-1} - \delta_\pi \tilde{\pi}_{t-1} - \delta_y \tilde{y}_{t-1}\right), \tag{9}$$

where the last term captures monetary policy inertia (Rudebusch, 2006). Equations 7-9 imply that $\tilde{M}_t$ follows $VAR(5)$ dynamics with coefficients given in the appendix. This special case has the same number of parameters as an unrestricted VAR(1) specification, which is a natural comparison that we also estimate below. In the results, our baseline model is based on these cross-equation restrictions from Laubach and Williams (2003). Section 3.6 compares the results obtained using a VAR(1) for $\tilde{M}_t$ and shows that the baseline cross-equation restrictions play an important role to obtain realistic estimates of the potential output and the output gap.

2.3 Identifying Nominal Shocks

The conditional mean specification in Section 2.2 implies restrictions on the matrix Σ in Equation 2:

$$\Sigma \equiv \begin{bmatrix} \sigma'_{\tilde{i}} \\ \sigma'_{\tilde{\pi}} \\ \sigma'_{\tilde{y}} \end{bmatrix} + \begin{bmatrix} \sigma'_{i^*} \\ \sigma'_{\pi^*} \\ \sigma'_{y^*} \end{bmatrix} = \tilde{\Sigma} + \Sigma^*, \tag{10}$$

and additional identification assumptions about Σ are needed to recover the shocks ε_t from the data. Since it must be that $\Omega = \Sigma\Sigma'$, we need an additional $N(N-1)/2$ restrictions as well as sign and ordering assumptions to identify the shocks (N is the size of matrix Σ). Kilian and Lütkepohl (2017) discuss identification in structural VAR models.

Nominal and real shocks The distinct feature of our approach that we leverage to identify nominal shocks is that the same shocks ε_t drive the secular and cyclical components of macro variables. The definition of the nominal shock that we use is standard. It is a shock that may influence cyclical fluctuations in the output, inflation and interest rates but leaves unchanged the output growth and real rate in the long run. This definition corresponds to the idea in structural models that a nominal shock does not influence the steady-state output and real rate that would prevail under flexible prices (i.e., the natural output and the natural real rate as in Gali 2008). Nominal shocks may capture monetary policy shocks as well as other shocks affecting the supply of inside money via banks or the financial system (Friedman and Schwartz, 1963; Brunnermeier and Sannikov, 2016), and we do not take a stand on what sources are more important.

Based on this definition of the nominal shock, we introduce the following restrictions on the variance parameters:

$$e_1'\sigma_{r^*} = 0 \qquad e_1'\sigma_{g^*} = 0 \qquad e_2'\sigma_{g^*} = 0, \tag{11}$$

where e_j is a vector of zeros but with element j equal to 1 and $\sigma_{r^*} = \sigma_{i^*} - \sigma_{\pi^*}$. While we defer a formal discussion until Section 3, we'll state for now that these restrictions are enough to identify Σ. The first two restrictions mean that (i) the nominal shock is ordered first and that (ii) it is uncorrelated with the unexpected changes in r_t^* and g_t^*. These two restrictions implement the definition of a nominal shock.

The last restriction deals with the second and third shocks. Both these shocks can drive changes to the secular and cyclical components of the output and real rate. Hence, we label them as real shocks. However, we do not attempt to distinguish between these two shocks. Instead, the last restriction simply says that the second shock is uncorrelated with unexpected changes in g_t^*. This is one of the possible orthogonalizations and has no economic content. For instance, it is not meaningful in an economic sense to distinguish between the impulse response functions with respect to these two shocks.

This identification of the nominal shock is close in spirit to the long-run restrictions of demand and supply shocks introduced by Blanchard and Quah (1989). Their identification strategy pins down a "demand" shock that has no long-run impact on unemployment and output as well as a "supply" shock that can have a long-run impact on output (see Dungey et al. (2015) for a recent analysis of long-run restrictions).

Caveats Blanchard and Quah (1989) analyze plausible circumstances in which the interpretation in terms of demand and supply shocks is valid but they discuss important caveats, which are worth repeating here. But note that, despite these caveats, we believe that our approach provides a useful perspective on the changing role of nominal and real shocks in the bond market.

One caveat is that the demand shock (or the nominal shock in our case) in the model may commingle a variety of actual demand disturbances with different dynamic effects on interest rates and output. This aggregation of different effects can influence the interpretation of impulse response functions. This caveat is an important reason why we use the label "nominal" instead of "demand" (as well as "real" instead of "supply"), since this aggregation may capture supply shocks that have little or no long-run impact.

Another caveat is that even demand disturbances may have a long-run impact on output. Demand disturbances that influence the saving rate may subsequently influence the long-run capital stock and output. Increasing returns or learning-by-doing would also mean that we may not be able to recover distinct demand and supply shocks using long-run restrictions. In this case, the identification of nominal shocks that we use would push these demand disturbances as part of the real shocks that we identify. This offers one interpretation of one of the two types of real shocks—the one that is uncorrelated with g_t^*—although there are other possibilities.

Even then, if demand has no long-term impact, its effects on capital accumulation may be indistinguishable from a truly permanent impact in finite-sample data. Blanchard (2018) voices this issue given the slow recovery that followed the 2008-2009 financial crisis, asking whether potential output is really independent of nominal disturbances (or monetary policy), and whether there really is no long-run trade-off between inflation and output. Jordà, Singh and Taylor (2020) provide international evidence that capital stock and total factor productivity exhibit hysteresis with respect to monetary policy shocks.

2.4 Time-Varying Loadings

The class of models discussed in Section 2.2 has constant loadings parameters $\bar{\Sigma}$, $\tilde{\Sigma}$ and σ_{y^*}, but we are mostly interested in the specifications with time-varying loadings. There are several reasons for this. First, the consensus is that π_t^* was more volatile during the 1970s and the 1980s relative to the period following the 1990s (See, e.g., Figure 2b in Stock and Watson 2016) but that r_t^* was relatively more volatile during the period after the mid-1990s. The existing estimates of π_t^* show a flat pattern starting sometimes after the mid-1990s. By contrast, the existing estimates of r_t^* show larger changes and a declining pattern after the mid-1990s. There is also ample evidence that the cyclical component of the short rate $\tilde{i}_t$ is much more sensitive early in recessions or around periods of financial distress (Cieslak and Povala, 2016). These patterns suggest that the sensitivities to macroeconomic shocks have changed over time. We interpret these as changes in the loadings of the shocks on the cyclical and secular components of the macro. This also means that the macro variables have time-varying volatilities.

Cyclical Components We consider an approach that is simple and parsimonious and which preserves the connections with the established identification strategies in the literature. The loadings of the cyclical components on the shocks are given by the following:

$$\tilde{\Sigma}_t = \begin{bmatrix} v_{\tilde{i},t} & 0 & 0 \\ 0 & v_{\tilde{\pi},t} & 0 \\ 0 & 0 & v_{\tilde{y},t} \end{bmatrix} \begin{bmatrix} \sigma'_{\tilde{i}} \\ \sigma'_{\tilde{\pi}} \\ \sigma'_{\tilde{y}} \end{bmatrix} = \tilde{V}_t \tilde{\Sigma}, \tag{12}$$

where the matrix $\tilde{\Sigma}$ is unrestricted up to some identification assumptions we discuss below. Each vector $\sigma_{\cdot}$ combines the shocks in fixed proportions while each scalar $v_{\cdot,t}$ controls the scale of the innovations to each of the cyclical components over time. The time-varying factors $v_{\cdot,t}$ have dynamics specified in Section 2.5 and are estimated jointly with other parameters of the model. Then, the cyclical components have dynamics given by:

$$\tilde{M}_t = \Phi(L)\tilde{M}_t + \tilde{\Sigma}_{t-1}\varepsilon_t.$$

Secular Components We follow a similar route for the loadings of the shocks on the secular components. For $\bar{M}_t$, these are given by the following:

$$\bar{\Sigma}_t = \begin{bmatrix} v_{i^*,t} & 0 & 0 \\ 0 & v_{\pi^*,t} & 0 \\ 0 & 0 & v_{g^*,t} \end{bmatrix} \begin{bmatrix} \sigma'_{i^*} \\ \sigma'_{\pi^*} \\ \sigma'_{g^*} \end{bmatrix} = \bar{V}_t \bar{\Sigma}, \tag{13}$$

where the matrix $\bar{\Sigma}$ is unrestricted. For the potential output, we have:

$$\sigma_{y^*,t} = v_{y^*,t}\sigma'_{y^*}, \tag{14}$$

but we restrict the scaling factor $v_{y^*,t} = v_{g^*,t}$. Again, each scalar $v_{\cdot,t}$ has dynamics given in Section 2.5 with parameters that are estimated jointly with the rest of the model. These scalar $v_{\cdot,t}$ varies the scale of the innovations to each of the secular components. However, the relative contribution of each shock is parameterized by the matrix $\bar{\Sigma}$. Then, the secular components have dynamics given by:

$$\bar{M}_t = \bar{M}_{t-1} + \bar{\Sigma}_{t-1}\varepsilon_t$$
$$\Delta y^*_t = g^*_{t-1} + \sigma'_{y^*,t-1}\varepsilon_t.$$

2.5 Loading Dynamics

The scaling factors must remain positive, and we use $EGARCH(1,1)$ dynamics for this purpose (see Nelson, 1991). To illustrate, in the case of the cyclical short rate $\tilde{i}_t$, we have that

$$\ln\left(v^2_{\tilde{i},t}\right) = \omega_{\tilde{i}} + \beta_{\tilde{i}} \ln\left(v^2_{\tilde{i},t-1}\right) + g\left(z_{\tilde{i},t}\right), \tag{15}$$

where $g(z_{\tilde{i},t})$ is the innovation to the variance process given by a function of the normalized variable $z_{\tilde{i},t} \equiv (\alpha'_{\tilde{i}}\varepsilon_t)/\sqrt{\alpha'_{\tilde{i}}\alpha_{\tilde{i}}}$. The scaling factors $v_{\tilde{\pi},t}$ and $v_{\tilde{y},t}$ for the inflation and output gaps, respectively, have similar dynamics. Therefore, the innovation is driven by the shocks ε_t, the loadings of each shock is given by the vector of parameters $\alpha_{\tilde{i}}$ and the denominator is a convenient normalization. All these parameters are estimated jointly. The function $g(\cdot)$ allows for asymmetry in the effects of $z_{\tilde{i},t}$ on $v^2_{\tilde{i},t}$, which is consistent with the evidence of the substantial asymmetry in the pattern observed in the output gap measures across the National Bureau of Economic Research expansion and recession phases (Morley and Piger, 2012).[3]

We introduce an important restriction to the process of time-varying loadings for secular components. The reason for this is that, in standard unobserved component models, the variance of the secular changes can diverge as the forecast horizon increases. This severely over-estimates the uncertainty around the future, which affects standard analytical tools like the variance's decomposition. To see what the issue is, consider the

[3]This function is given by $g\left(z_{\tilde{i},t}\right) = \sqrt{\alpha'_{\tilde{i}}\alpha_{\tilde{i}}} z_{\tilde{i},t} + \kappa_{\tilde{i}}\left(\left|z_{\tilde{i},t}\right| - E\left[\left|z_{\tilde{i},t}\right|\right]\right)$.

variance of $i^*_{t+\tau}$ some τ periods ahead:

$$Var_t\left[i^*_{t+\tau}\right] = \left(\sigma'_{i^*}\sigma_{i^*}\right)\left(\sum_{j=1}^{\tau} E_t\left[v^2_{i^*,t+j-1}\right]\right),$$

where the coefficient for each term in the sum is equal to 1 because of the unit root in the process for i^*. Proposition 1 provides sufficient conditions on the properties of $v^2_{i^*,t}$ for the convergence of $\lim_\tau Var_t\left[i^*_{t+\tau}\right]$.[4]

Proposition 1 *Suppose the scalar $v^2_{i^*,t}$ has $EGARCH(1,1)$ dynamics given by Equation (15), where $z_{i^*,t} \equiv (\alpha'_{i^*}\varepsilon_t)/\sqrt{\alpha'_{i^*}\alpha_{i^*}}$ and*

$$g\left(z_{i^*,t}\right) = \sqrt{\alpha'_{i^*}\alpha_{i^*}} z_{i^*,t} + \kappa_{i^*}\left(|z_{i^*,t}| - E\left[|z_{i^*,t}|\right]\right).$$

If $\beta_{i^*} = 1$ and $\omega_{i^*} < \bar{\omega}_{i^*}$, which is defined in the online appendix, then $Var_t\left[i^*_{t+\tau}\right]$ converges,

$$Var_t\left[i^*_{t+\tau}\right] \underset{\tau}{\longrightarrow} \frac{\left(\sigma'_{i^*}\sigma_{i^*}\right)}{\theta_{i^*}} v^2_{i^*,t},$$

where $\theta_{i^*} \equiv 1 - e^{\omega_{i^*}-\bar{\omega}_{i^*}}$ and $\Phi_N(\cdot)$ is the standard normal distribution's cumulative distribution function.

The restrictions on the $EGARCH(1,1)$ guarantee that $Var_t\left[i^*_{t+\tau}\right]$ converges to a positive scalar for arbitrarily large horizons τ. Intuitively, the shocks that are very far away in the future eventually have no impact on the secular component (in expectations). Stock and Watson (2007) and Wright (2011) analyze unobserved component models with stochastic volatility (for the case of inflation), and the volatility dynamics that they use are obtained by imposing $\omega = 0$, $\beta = 1$ and $\kappa = 0$, which implies that $Var_t\left[\pi^*_{t+\tau}\right]$ diverges with the horizon in the model, since $\omega = \bar{\omega} = 0$. Absent the restriction that we introduce, the

[4] This approach can be generalized to other processes. A similar result is available from the authors in the context of the asymmetric $GARCH(1,1)$ dynamics.

volatility of any of the variables influenced by the secular components would diverge as the forecast horizon increases, which would severely over-estimate the uncertainty and affect such standard analytical tools as the decomposition of the variance.

2.6 Bond Prices

We build on the assumption of no-arbitrage, which guarantees the existence of a positive pricing kernel process ξ_{t+1} such that the price of any asset V_t that does not pay any dividends at time $t+1$ satisfies $V_t = E_t[\xi_{t+1} V_{t+1}]$. The online appendix specifies the pricing kernel and the prices of risk as well as the pricing equation for bonds.

To summarize, we assume that the macro shocks are priced in the bond market so that the macro variables enter the bond pricing equation. However, we impose that bond expected returns are stationary. In addition, we follow Ang and Piazzesi (2003) and others in introducing one additional latent source of risk that is uncorrelated with macro shocks. This new risk drives a latent factor f_{t+1} meant to capture the residual variations in nominal yields that are unexplained by the macro variables. We label f_t a term structure factor and construct it such that it does not influence the dynamics of the short rate. For this purpose, it has simple AR(1) dynamics given by:

$$f_{t+1} = \mu_f + \phi_f f_t + u_{f,t+1}, \tag{16}$$

where the innovation is given by $u_{f,t+1} = \sigma'_{fM} \tilde{\Sigma} \varepsilon_{t+1} + \sigma_f \varepsilon_{f,t+1}$, and where $\varepsilon_{f,t+1} \sim N(0,1)$ is uncorrelated with the macro shocks. Using standard arguments, we then show that the bond yields are given by:

$$Y_t^{(n)} = i_t^* + a_n + b'_n \tilde{M}_t + b_{f,n} f_t, \tag{17}$$

for $n > 0$ with initial conditions $a_1 = 0$, $b_1 = e_1$ and $b_{f,1} = 0$ so that $Y_t^{(1)} = i_t = i_t^* + e'_1 \tilde{M}_t$

and where the coefficients are given by the following:

$$
\begin{aligned}
b_n' &= (1/n)\, e_1' \left(I - (\Phi_Q)^n\right) (I - \Phi_Q)^{-1} \\
b_{f,n} &= \left[b_n' - (1/n)\, e_1' \left((\phi_{Q,f})^n I - (\Phi_Q)^n \right) (\phi_{Q,f} I - \Phi_Q)^{-1}\right] \frac{\Phi_{Q,Mf}}{1 - \phi_{Q,f}},
\end{aligned}
$$

and the constant is given by this recursion:

$$
\begin{aligned}
(n+1)\, a_{n+1} = n a_n + n b_n' K_{Q,0} - \frac{n^2}{2} (\sigma_{Q,f})^2 (b_{f,n})^2 \\
- \frac{n^2}{2} e_1' \bar{\Sigma} \bar{\Sigma}' e_1 - \frac{n^2}{2} b_n' \tilde{\Sigma} \tilde{\Sigma}' b_n - n^2 e_1' \bar{\Sigma} \tilde{\Sigma}' b_n,
\end{aligned}
$$

where parameters with the subscript Q characterize the risk-neutral measure and must be estimated with other parameters (see the online appendix for details).

Equation 17 implies that the constant term captures the average term premium (e.g., this term is 1.66 percent for the 10-year yield in our results). Note also that bond yields share one common cointegration relationship. This makes the model more parsimonious. This also guarantees that any interest rate spread are stationary. The same is true about expected returns from holding bonds. Hall, Anderson and Granger (1992) provide early evidence that bond yields are cointegrated and that the bond risk premium is stationary (see also Engsted and Tanggaard, 1994). Consistent with this observation, Cieslak and Povala (2015) find that the predictability of bond excess returns based on current yields essentially doubles once the inflation trend is removed from these yields. Bauer and Rudebusch (2020) emphasize that describing bond yields as cointegrated processes in a dynamic term structure model is useful in small samples. Finally, our approach implies multiple unspanned volatility factors and is similar to the earlier works by Creal and Wu (2017) and Hansen (2019) that analyze macroeconomic risk in bond yields.

3 Data and Results

We estimate several versions of the model with a focus on understanding nominal shocks. The online appendix provide the restrictions that we implement for parameter identification as well as the likelihood of that data that we use for estimation. Our baseline model incorporates the cross-equation restrictions given in Section 2.2 as well as time-varying loadings for shocks. We also estimate the case with constant loadings, which is nested in the baseline model. We find that letting the shock loadings vary over time is important to capture how the impact of nominal shocks changes over time. We also estimate the case with unrestricted VAR(1) dynamics. We find that the cross-equation restrictions from Laubach and Williams (2003) help pin down the output gap and potential output and that the VAR(1) estimates are implausible, for instance when compared with the Congressional Budget Office (CBO) estimates.

3.1 Data

Macro Data We estimate the model using US data. The sample period starts in 1983 and it ends in 2019. The sampling frequency is quarterly. Panel (a) of Figure 1 shows the macroeconomic data. For the short rate i_t, we use the secondary market rate for 3-month Treasury bills, in percentage terms. For the inflation rate π_t, we use the compounded rate of change of the personal consumption expenditures index, excluding food and energy, annualized, in percentage terms and seasonally adjusted. For output y_t, we use real gross domestic product, in log of billions and seasonally adjusted. We use the macro data available from the Federal Reserve Bank of St. Louis website.

Yield and Survey Data Panel (b) of Figure 1 shows the yields for bonds with 2-, 5- and 10-year maturities. For our estimation, we select the zero-coupon yields with annual maturities between one and ten years, in annualized percentage terms, from the GSW database (Gurkaynak, Sack and Wright, 2006). We also use data about the rates of inflation swaps with 2-, 5- and 10- year maturities.

In the spirit of Kim and Orphanides (2012), we use survey data to help with the precision of the estimation. Long-horizon survey forecasts can improve the identification of secular and cyclical components. Orphanides and Wei (2012) also use surveys as a way to capture structural changes in a stationary VAR that they estimate recursively. Cieslak and Povala (2015) find that using survey-based expectations of inflation from a variety of sources to control for expected inflation in predictive regressions of bond excess returns consistently strengthens the return predictability.

Using survey data plays an important role in disciplining the model. The Beveridge-Nelson decomposition relies on the difference in the persistence between components. In practice, it is difficult to pin down this difference in a short sample when using only the likelihood. Intuitively, increasing the variance of the secular changes may improve the fit of the one-period forecasts, which is the basis of the likelihood, but this can also cause over-fitting and produce secular component estimates that are implausible. However, at very long horizons, the model forecasts depend on the secular components but not on the cyclical components. Hence, long-term surveys help to identify variations in the secular components as well as obtain stable and sensible estimates of the secular components.

We use estimates of the long-horizon forecasts for the short rate, inflation and GDP growth available from Crump, Eusepi and Moench (2016). The idea behind their estimates is that different surveys offer forecasts at different frequencies and horizons, which can be pooled together to obtain smoothed estimates at regular frequencies and fixed horizons. Clearly, the measurement errors surrounding the smoothed estimates are not negligible, and this uncertainty will be embedded in the estimation procedure, which we discuss below. Therefore, different model specifications may deliver a range of plausible but different estimates, even when survey data are used.

Panel (c) of Figure 1 shows long-horizon survey forecasts for the short rate, inflation and output growth in our sample period. The secular changes are visually apparent. There is a slow decline for inflation, which exhibits a plateau starting at around 1998.

There is also a slow decline for the short rate, as well as a plateau starting at around 1998, but this decline resumes after 2009. This renewed decline is absent from the survey forecasts for inflation and has been attributed to a decline in the real neutral rate. Finally, the long-horizon forecasts show output growth rising to 3 percent for much of the decade after 1998 but declining to below 2 percent by the end of our sample period.

3.2 Real and Nominal Shocks

The shocks that we recover, especially the nominal shocks, are key outputs of the model. Panel (a) of Figure 2 shows the shocks in our baseline model and Panel (b) reports the shocks in a model with constant loadings. The results from both models suggest that nominal shocks were prevalent in every recession in our sample but that they have been less prevalent during the protracted recovery since 2009.

However, there are some differences among the shocks from these two models. To gauge the differences, Panels (c)-(e) of Figure 2 show scatter plots for each type of shocks in these two models (the units in the scatter plots are standard deviations). The correlations range from 0.61 and 0.72, for the two real shocks, and up to 0.79 for the nominal shocks (this translates in univariate regression R^2s of 37, 52 and 62 percent, respectively). Some of the differences are large. One notable example is the large negative shocks late in 2008. The negative nominal shock is much larger in the model with time-varying loadings, while the real shocks play the bigger role in the model with constant loadings. This difference corresponds to the outlier that we can see to the left and bottom of Panel (e).

Overall, this shows that the assumption of constant loadings influences the identification of the nominal and real shocks. In the following section, we show that this leads to different interpretations of some of the cyclical and secular changes in interest rates. In this particular case, using constant loadings leads to the conclusion that the great financial crisis had a large secular impact on inflation, the real rate and, therefore, on bond yields.

3.3 Endpoint Estimates

The endpoints π_t^* and r_t^* play an important role in "de-trending" bond yields within the model, as in Cieslak and Povala (2015) and Bauer and Rudebusch (2020). This section shows that assuming time-varying or constant shock loadings leads to different endpoint estimates. More importantly, using time-varying loadings reveals different nominal shocks.

Inflation Endpoint Figure 3 presents the inflation endpoint π_t^* from the baseline model and the model with constant loadings. The overall pattern is unsurprising. Early in our sample, the inflation endpoint was around 6 percent, but it quickly fell and remained on a plateau of around 4 percent for most of the 1980s. Starting in 1992, the endpoint of inflation gradually declined again and reached a lower plateau of slightly above 2 percent around 1998; it stayed there for the remainder of the sample. The models closely agree with each others but the baseline model produces a much smoother path because of the time-varying loading $v_{\pi^*,t}$. To see the effect of this loading, Panel (b) of Figure 3 shows the time-series of the one-year-ahead volatility for the shifting endpoint $\text{var}_t(\pi_{t+4}^*)$. This shows that, in our sample, the inflation endpoint became anchored. Early on, the annual volatility was around 30 basis points, but it dramatically declined after 1990, reaching a very low level of between 1 and 3 basis points after 2000 (for comparison, the standard deviation of the observed inflation rate is 52 basis points in our sample). In the model with constant volatility, the conditional variance $\text{var}_t(\pi_{t+4}^*)$ is constant at 23 basis points, which is why the endpoint estimate continues to fluctuate in this model throughout our sample period.

One key benefit of our framework is that we can document the contribution of nominal shocks to the inflation endpoints π_t^*, shown in Panel (c) of Figure 3. We also report the total contribution of both real shocks. The baseline results show that both nominal and real shocks influence secular inflation. Overall, across the sample, around two thirds of the decline is attributed to the cumulative impact of the nominal shocks. The baseline results also show that the impact of shocks decline toward zero during the sample, which is due to changes in the $v_{\pi^*,t}$ loading. Early in the sample, the loading of nominal shocks is around 0.1, meaning that π_t^* tended to absorb one tenth of the nominal shocks during that period, but this loading declines toward zero over the course of our sample. The real shocks loadings exhibit similar changes because of changes to $v_{\pi^*,t}$.

How the shock loadings change over time plays an essential role to capture the anchoring of π_t^*. To see this, we report in Panel (d) the contribution of each shock in the model with constant loadings. In this restricted case, nominal shocks play a much smaller role throughout the sample. In fact, most of the disinflationary period until the mid-1990s is attributed to real shocks in this model. Overall in the sample, around 60 percent of the decline in π_t^* is attributed to the real shocks. This model interpretation clashes with the general view that the secular inflation decline during this period was largely due to the actions of central banks. In addition, real shocks in this model continue to cause substantial updates to π_t^* throughout the sample. This is inconsistent with the general view that secular inflation variations are very small toward the end of the sample. For instance, the episode of the great financial crisis shows a large negative impact of real shocks on π_t^* but that reverted after only a few years.

Real-rate Endpoint Panel (a) of Figure 4 presents the estimates of the real-rate endpoint r_t^*. The overall pattern shows three phases that are broadly in line with the results reported by Laubach and Williams (2003) and Holston, Laubach and Williams (2017) for the US. First, r_t^* exhibits a gradual decline over the ten years between 1983 and 1993, ending soon after the 1990-1991 recession. This is a period when the decline in the long-term forecasts of the short rate was more pronounced than the decline in the forecasts of inflation. Then, the r_t^* estimate rises from 1993 to 2000, which corresponds to the acceleration in productivity growth predicted by Fed chairman Greenspan. During this period, output growth accelerated but inflation remained subdued, which was attributed to the long-awaited impact of information technology on productivity. Meyer (2003) offers a detailed review of the policy discussions that took place during that period. However, part of the decline in r_t^* may be due to real benefits from the lower level and lower volatility of inflation (see, e.g., Barro (2013)). After this episode, the neutral rate resumed its decline, reaching 2 percent around 2008 and 0.5 percent in our baseline model at the end of our sample.

Our results shed some light on the volatility of the neutral real rate. Panel (b) shows that the neutral rate's one-year-ahead volatility is 14 basis points in the model with a constant volatility. By contrast, the estimated volatility of the real-rate endpoint in the baseline model is less than 5 basis points after 2005. Therefore, the decline in the baseline real-rate endpoint estimates after 2008 is due to a succession of correlated shocks that push r_t^* in the same direction and not to the incidence of a few large shocks.

Panel (c) reports the contribution of both real shocks separately (nominal shocks play no role here). We find that the first type of real shocks plays a greater role early in the sample but that the second type of real shocks becomes more important during the 2000s. The first type is orthogonal to the innovations in g^*, while the second real shock can influence both r^* and g^*. Hence, changes to the secular real rate tended to be uncorrelated to changes to the potential growth g_t^* in the first half of the sample up until roughly 1998.[5] However, the recent decline in r_t^* after 2008-2009 is largely driven by shocks correlated with changes to potential shocks, suggesting that technology, capital accumulation and demographics played a larger role during that period. Finally, Panel (d) reports the contribution of real shocks in the model with constant loadings. The difference is stark. The model with constant loadings shows no changes in the nature of the real shocks that

[5]This raises the question of whether the variations in r_t^* early in our sample can be interpreted as variations in the neutral real rate, because standard models would associate changes to the neutral real rate to changes in potential output growth. We leave the analysis for future research whether further economic restrictions on the identification of shocks to the neutral rate.

are driving r_t^* and no compression in the dispersion of these shocks.

3.4 Impulse Response Functions

The disappearing impact of nominal shocks on π_t^* coincides with changes in the impulse response functions of every observed macro variable. We find that the responses of the short rate, inflation and output to nominal shocks decrease between the beginning and the end of the sample. To see the changes, Figure 5 compares impulse response functions of the short rate, inflation and output at two dates in time. One date is the first quarter of 1984, which is in the period when Figure 3 shows an important disinflationary role for nominal shocks. The other date is the first quarter of 2007, which is in the period when Figure 3 shows that the impact on secular inflation is tiny. We choose 2007Q1 because this precedes the great financial crisis and avoids the extended period that follows, when the 3-month rate hovered just above zero. This way, the changes that we report cannot be associated with potential changes due to the lower bound below nominal yields. Focusing on two dates makes it easy to see the contrasts. Choosing different dates in these two periods does not change the message.

The results in Figure 5 show that, early in the sample, a nominal shock leads the short rate to increase by 0.45 percentage points on impact and peak at 0.6 points after two years. This is associated with an increase in the inflation rate of around 0.2 percentage points, which is essentially permanent, and an increase in output growth of around 0.5, which is short-lived. One interpretation of these results is that, when inflation is unanchored, nominal shocks lead to the economy over-heating, a persistent increase of inflation along with a large and persistent policy rate response. However, the responses are very different later in our sample. The impact on the short rate peaks at only 0.2 percentage points and is less persistent, the impact on inflation is large but quickly disappears, and the impact on output growth is small and also quickly disappears. One interpretation is that, with inflation anchored, nominal shocks may cause substantial inflation, but this increase is transitory, with little economic overheating and only a small short rate response. This pattern is consistent with the theoretical argument in Clarida, Gali and Gertler (2000) that the post-1979 policy responses to inflation stabilized output and inflation. It is also consistent with the empirical results in Cogley and Sargent (2005) that policy activism by the Federal Reserve after the chairmanship of Paul Volcker contributed to the fall of inflation as well as to changes in its persistence. Since it is plausible that monetary policy shocks are captured by the nominal shocks, the results seem to be consistent with the conclusion by Ramey (2016), in a review of the empirical evidence, that monetary policy has become more systematic over time.[6]

3.5 Bond Yields

Given the substantial changes in the macroeconomic impulse response functions with respect to nominal shocks during our sample period, there should be no surprise that we find that the response of bonds to nominal shocks also change.

Bond Yield Impulse Response Function Figure 7 shows the impulse response functions of bond yields with 3-month, 2-year and 10-year maturities with respect to nominal shocks. We show the responses at these three maturities on two dates, as in the previous section. The first row exhibits the responses of the bond yields, the expectation components and the term premium for the first quarter of 1984. The second row exhibits the same responses but for the first quarter of 2007.

Early in the sample, the initial impact on the short rate is around 0.45 percentage points and peaks close to 0.6 (which are the same as in Panel (a) of Figure 5). The initial impact on the 2-year yield is around 0.7 percentage points and peaks close to 0.8, while the initial impact on the 10-year yield is around 0.6 percentage points and peaks close to 0.7.

[6] In the online appendix, we report a decomposition of the cyclical short rate $\tilde{i}_t$ in terms of nominal and real shocks. The results confirm that nominal shocks played a large role throughout the first half of the sample but disappeared from $\tilde{i}_t$ variations afterward.

Therefore bond yields strongly respond to nominal shocks in the first half of the sample. Nominal shocks lead to (i) a higher level of bond yields, (ii) a steeper slope between the short rate and longer-term yields and (iii) an inversion of the yield curve between 2-year and 10-year maturities.

Panel (c) shows that the expectation component dominates the response of bond yields to nominal shocks. This is consistent with the interpretation that, when inflation is unanchored, nominal shocks lead to strong policy responses. The initial impact on the expectation component is around 0.6 and 0.4 percentage points for the 2-year and 10-year bonds, respectively, which explains the yield curve inversion. This impact gradually declines afterward but some of it is permanent because nominal shocks lift π_t^* (compare with Figure 6b). The impact on the term premium component is smaller, around 0.1 and 0.2 percentage points for the 2-year and 10-year bonds, respectively, and peaks close to 0.25 and 0.4.

This inversion between the 2- and 10-year bond yields arise because the response of the short rate increases with the horizon until after the responses of growth and inflation rates decrease. Note that the inversion predicts slower growth in the baseline model. The observation that the expectation component drives the predictive content of a yield curve inversion for future growth is consistent with the results by Ang, Piazzesi and Wei (2006).

The second row in Figure 7 shows the response on 2007Q1, which is typical for the second half of our sample. Stated simply, the responses of bond yields to nominal shocks are muted. The impact on the 2-year yield is close to 0.4 percentage points, which is half the peak impact in 1984Q1, and it is more quickly reversed. We find that the responses of the expectation components drive the more muted responses of bond yields. The impacts on the term premium are still substantial but dissipate faster than during the earlier period.

Term Premium Panel (a) of Figure 8 shows the term premium for the 10-year zero-coupon yield estimated in our model compared with the estimates of Adrian, Crump and Moench (2013) (ACM hereafter), available from the Federal Reserve Bank of New York, as well as the estimates of Crump, Eusepi and Moench (2016) (CEM hereafter). The ACM estimate is widely cited by analysts and practitioners. While ACM do not explicitly account for secular variations in i_t^*, the model specification and estimation procedure deliver term premium estimates with a smaller trend than the estimates from a typical maximum likelihood estimation. The CEM estimate relies on a rich panel of survey data to correct for the influence of secular variations in i_t^* and does not rely on the no-arbitrage restrictions.

Figure 8 shows that the three estimates share broad similarities, which is of course reassuring. There are two differences worth underlining. First, our baseline term premium estimate appears stationary. By contrast, both the ACM and CEM estimates show a downward trend from 4 percent at the beginning of our sample period to less than 0 percent at the end of our sample period. Hence, these estimates attribute a greater share of the high interest rates early in our sample to a higher term premium. Second, our baseline estimate exhibits stronger cyclical patterns throughout the sample. For instance, the baseline results exhibit a sharp drop in the term premium starting in third quarter of 2008 at the peak of the crisis, while the drop is more modest based on other models. While there are several differences in the purposes and assumptions underlying these three estimates, the key difference to explain these two observations is the combination of (i) allowing for secular changes in bond yields and (ii) the restriction embedded in the baseline estimates that the term premium is stationary.

Perhaps unsurprisingly, the term premium shocks is the most important driver of the term premium. Panel (b) reports the impulse response function of the 2-year and 10-year term premia relative to the shock $\varepsilon_{f,t+1}$. This is substantially larger than the impact reported in Figure 7 for nominal shocks. However, the effect dies out relatively quickly. For the 10-year bond, a one-standard-deviation term premium shock raises the yield by close to 0.6 percentage points over one quarter, which dissipates to around 0.4 and 0.2 after one and two years, respectively.

To illustrate the magnitude of this impact relative to all other macroeconomic shocks, Panel (c) of Figure 8 shows the share of the one-year-ahead variance of the term premium that is explained by each shock. This share changes over time because of the time-varying loadings. Overall, the term premium shock almost always explains two thirds or more of the term premium variations at this horizon. This is consistent with the common observation in the literature that macroeconomic variables explain a small share of the term premium variability. Nonetheless, and perhaps unsurprisingly, the macro shocks are more important than the term premium shock when monetary policy changes its target rate more actively. The share attributed to macroeconomic shocks reaches beyond 50 percent in a handful of cases, for instance around the 1990 recession, during the recovery after the 2001 recession and in 2008.

Panel (d) shows the variance decomposition for longer horizons, up to 40 quarters and fixing the date to 2007Q1. The share of the term premium variance attributed to macroeconomic shocks increases to 50 percent at horizons between 2 and 3 years and rises beyond that. At these longer horizons, the real shocks together explain the majority of term premium variations while, as expected, nominal shocks explain only a small share. This pattern is common across the sample. The real shocks typically dominate term premium variations at lower frequencies. Given that the term premium only depends on cyclical variables $\tilde{i}_t$, $\tilde{\pi}_t$ and $\tilde{y}_t$, this immediately implies that, based on our identification strategy (see the discussion in Section 2.3), real shocks have more persistent impacts on business cycle variations than nominal shocks for most of our sample.

3.6 VAR(1) Dynamics

One important question is whether the cross-equation restrictions in Section 2.2 play an important role in the results. Figure 9 reports the key results that highlight the differences. Panel (a) shows estimates of the potential output from our baseline model, from the model with constant loadings and from the model with constant loadings and with VAR(1) dynamics. The VAR(1) produces a potential output that is much smoother than in the other models. This is not innocuous, since it affects the output gap estimates and, indirectly, the shocks recovered from the model.

To see this, Panel (b) shows the output gap from the same models as well as the difference between real GDP from and the potential output from the CBO. The estimates from the baseline model and the model with the constant loadings closely agree with each other. By the end of the sample period, the estimated output gap is around -1 percent and the CBO estimate is close to zero. We also find that the CBO output gap is not as deep following the 2001 and 2009 recessions and tends to recover more quickly. We should expect some differences between these two estimates since they use different methods: the CBO estimate relies on the "production-function" method while the approach here instead relies on both cross-equation restrictions and the statistical properties of the secular versus the cyclical components (following Blanchard and Quah 1989). Coibion, Gorodnichenko and Ulate (2018) provide a detailed discussion of different approaches to estimating potential output and their relative advantages. Like us, they find that CBO estimates imply a quicker decline in the output gap than the results obtained when following Blanchard and Quah (1989). However, our results and the CBO results agree on the timing of the peaks and troughs. All these estimates indicate a largely U-shaped recovery toward potential output. They also indicate that the slowdown in potential growth started before 2008. These results are consistent with the evidence on the great recession presented by Eo and Morley (2020) based on a regime-switching model for US output.

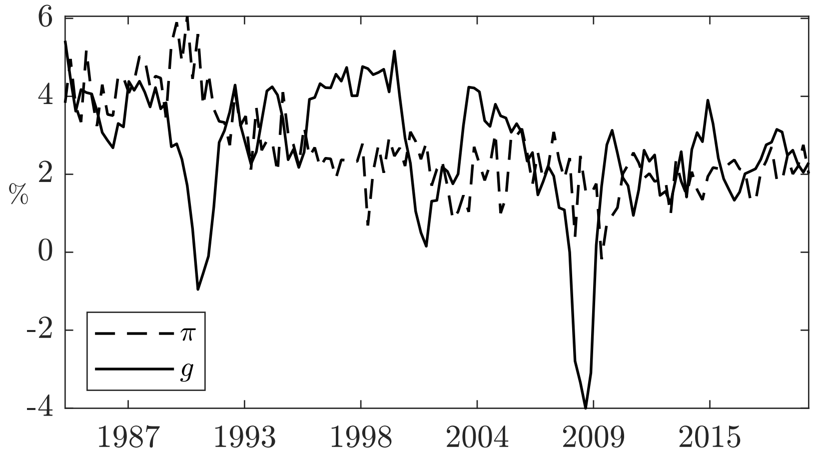
The output gap estimates based on the VAR(1) are very different, which suggests that the cross-equation restrictions play an important role in identifying the output gap. In fact, the estimates are implausible. The estimated output gap is between 1 and 5 percent during the period between 1997 and 2007, which includes the 2001 recession. Moreover, the estimated output gap based on the VAR(1) reaches -10 percent after 2009 and remains close to this level toward the end of our sample, in 2018. Hence, the results show that the restrictions from Laubach and Williams (2003) that we implement in our baseline model are useful for pinning down the output gap and the potential output, to a large extent because the Equation 8 connects the output gap to the monetary stance $\tilde{r}_t$. Note that these are relevant for applications when the system includes unobserved cyclical and secular components, as in our case. The results do not bear on the widespread use of the reduced-form VAR(1) dynamics to capture the relationship between observed macro variables when there are no unobserved components.

4 Conclusion

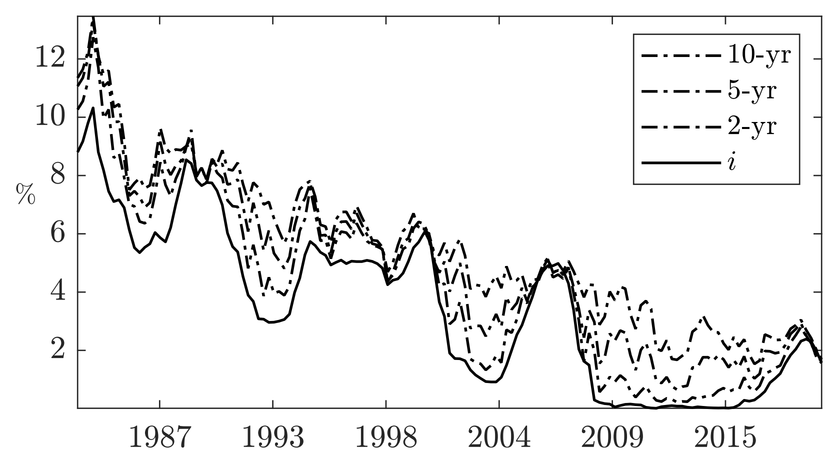
We provide a framework where secular and cyclical changes to the short rate, inflation and output are jointly determined by a small set of shocks with loadings that change over time. This framework incorporates pricing equations for bond yields in terms of macro variables in addition to a latent term structure factor. We use restrictions on the correlations between innovations to the secular and cyclical changes to identify and analyze the impact of nominal shocks. We find that the role and impact of nominal shocks on macro variables as well as on bond yields dramatically changed during our sample period. Our approach paves the road to identify the distinct impact of other types of structural shocks on the cyclical and secular changes in bond yields. In particular, our baseline model ignores the impact of regulatory or fiscal changes and does not distinguish between the roles of technology, demography and capital accumulation behind the evolution of potential output.

Figure 1: US Macro, Bond and Survey Data

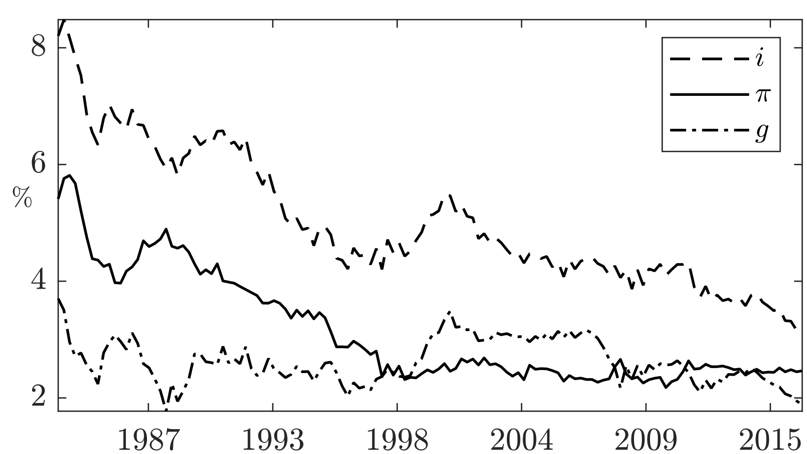
Panel (a) shows the quarterly inflation rate π_t as the compounded rate of change of the personal consumption expenditures index excluding food and energy, annualized, in percentage terms and seasonally adjusted as well as the output growth rate g_t as the log-change of seasonally adjusted real gross domestic product. Panel (b) shows the nominal three-month short rate i_t and the 2-, 5- and 10-year nominal zero-coupon rates (other maturities are not reported for clarity). Panel (c) shows long-horizon survey forecasts of i_t, π_t and g_t obtained from Crump, Eusepi and Moench (2016).



(a) Output growth and inflation data



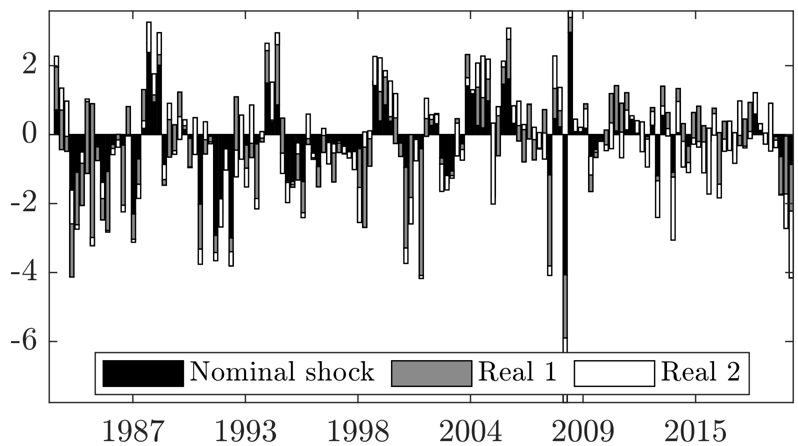
(b) Yield data



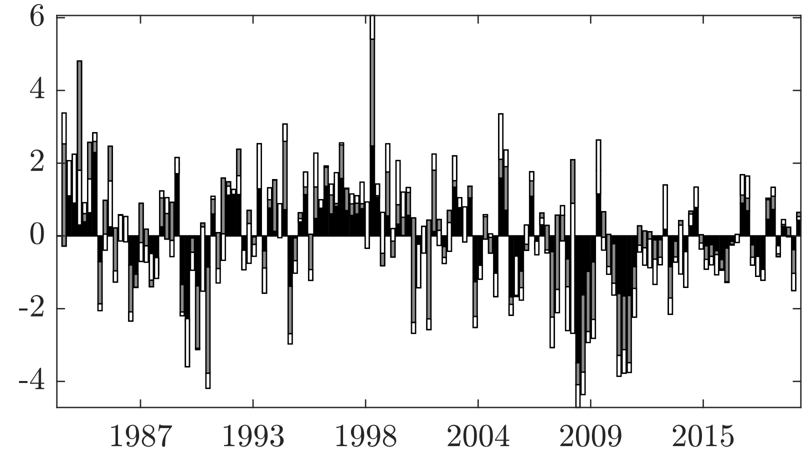
(c) Survey data

Figure 2: Estimated Real and Nominal Shocks

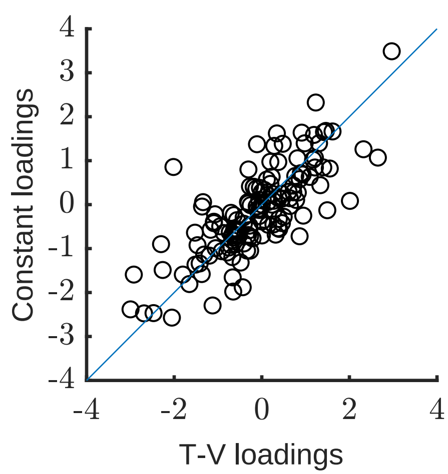
Nominal and real shocks identified in a model with time-varying loadings (Panel a) and constant loadings (Panel b). Panels (c)-(e) show scatter plots of the nominal shock and both real shocks with time-varying and constant loadings (x-axis and y-axis, respectively). Quarterly data, 1983–2019. Units are multiples of standard deviations in every panel.



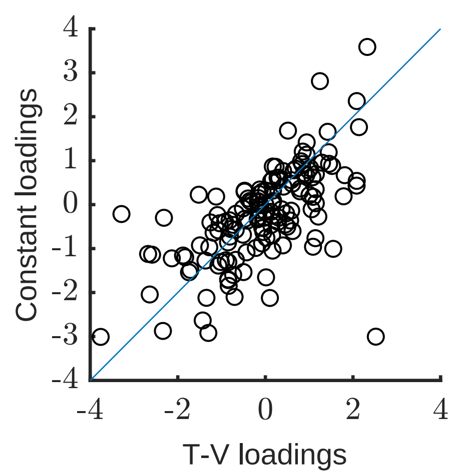
(a) Structural shocks (T-V loadings)



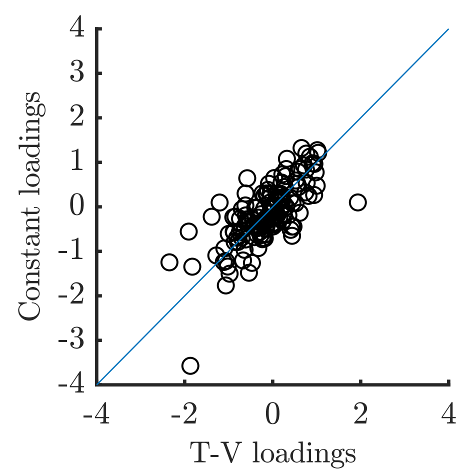
(b) Structural shocks (constant loadings)



(c) Nominal Shocks



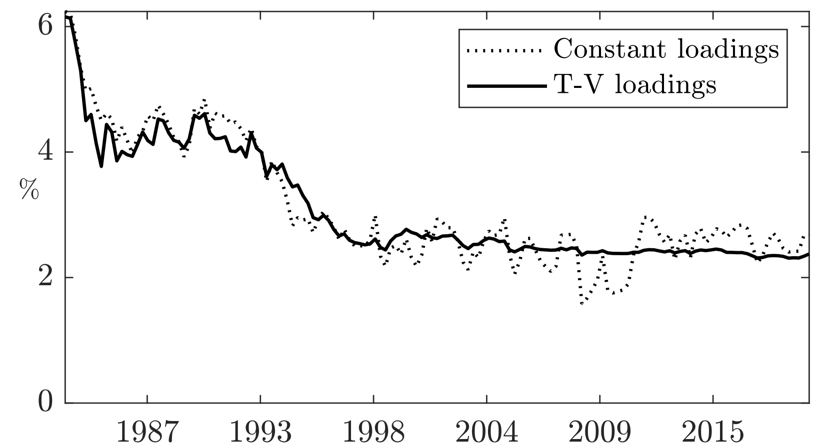
(d) Real Shocks 1



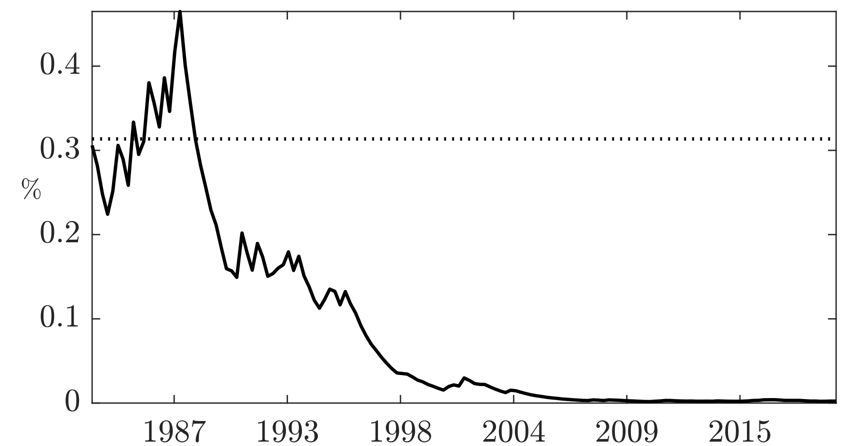
(e) Real Shocks 2

Figure 3: Inflation Endpoint π_t^*

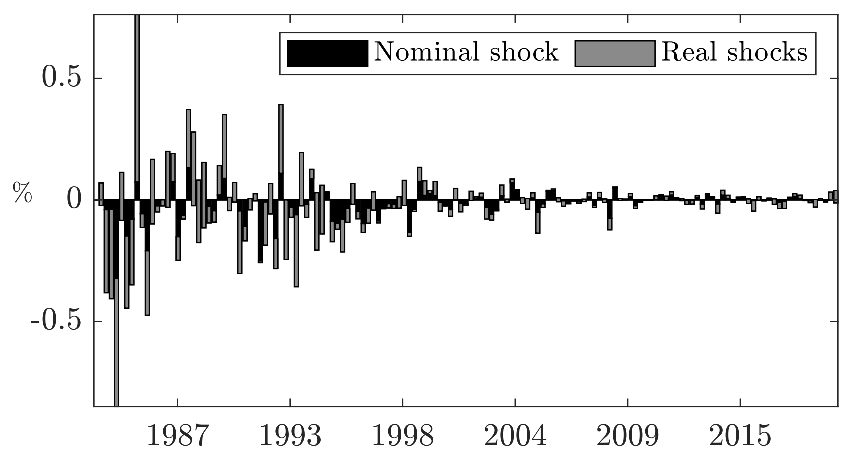
Panel (a) shows two series of inflation endpoint estimates π_t^* from models with constant or time-varying loadings (the baseline model), respectively, in percent. Panel (b) shows two series of estimated one-year-ahead conditional volatility of π_t^* from the same models, in percent. Panels (c)-(d) show the contribution of each shock to the period-by-period update of π_t^* in models with time-varying or with constant loadings, respectively, where the units are standard deviations.



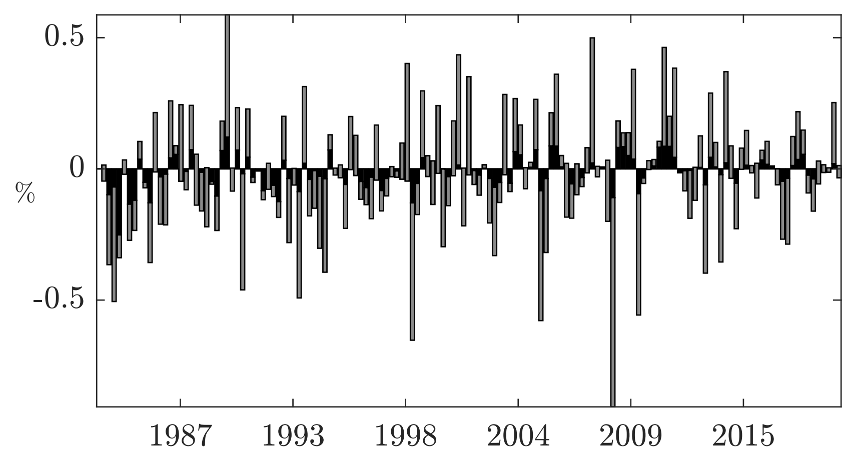
(a) Inflation endpoint π_t^*



(b) Inflation endpoint—Volatility



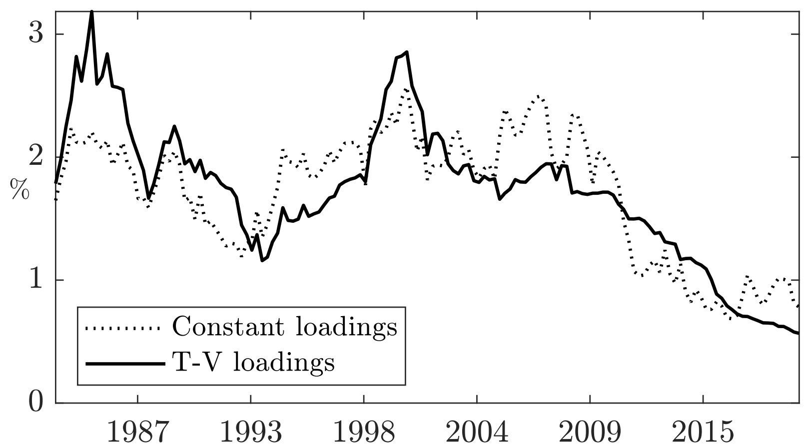
(c) Inflation endpoint—Shocks (T-V loadings)



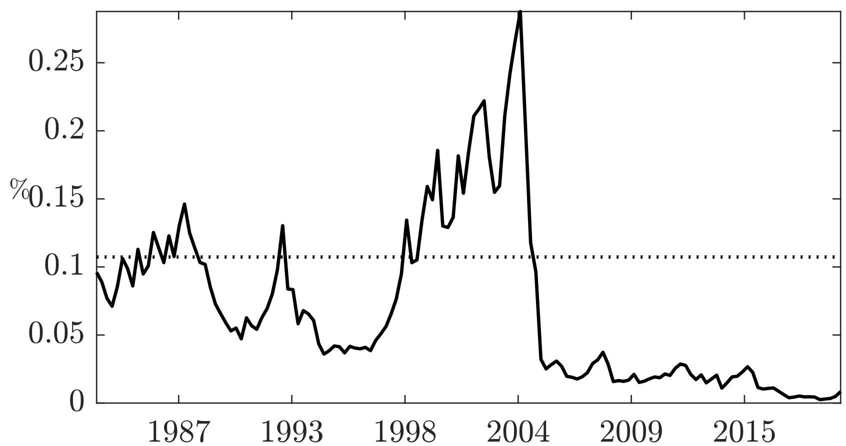
(d) Inflation endpoint—Shocks (constant loadings)

Figure 4: Real-rate Endpoint r_t^*

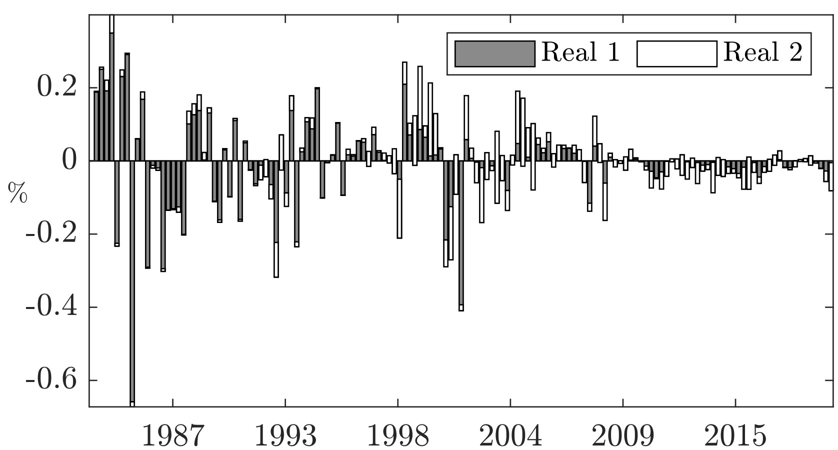
Panel (a) shows two series of real-rate endpoint estimates r_t^* from models with constant or time-varying loadings (the baseline model), respectively. Panel (b) shows estimated one-year-ahead conditional volatility of r_t^* from the same two models. Panels (c)-(d) show the contribution of real shocks to the period-by-period update in these two models (nominal shocks have no impact by construction). All units are annualized percentages.



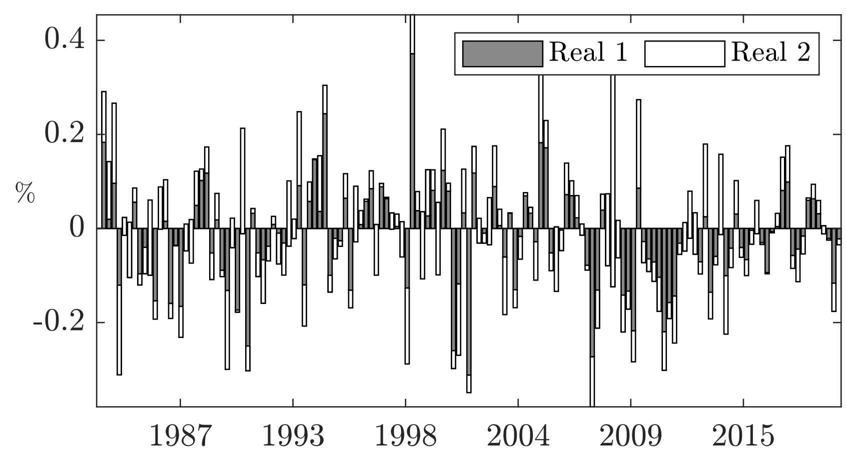
(a) Real-rate endpoint r_t^*



(b) Real-rate endpoint—Volatility



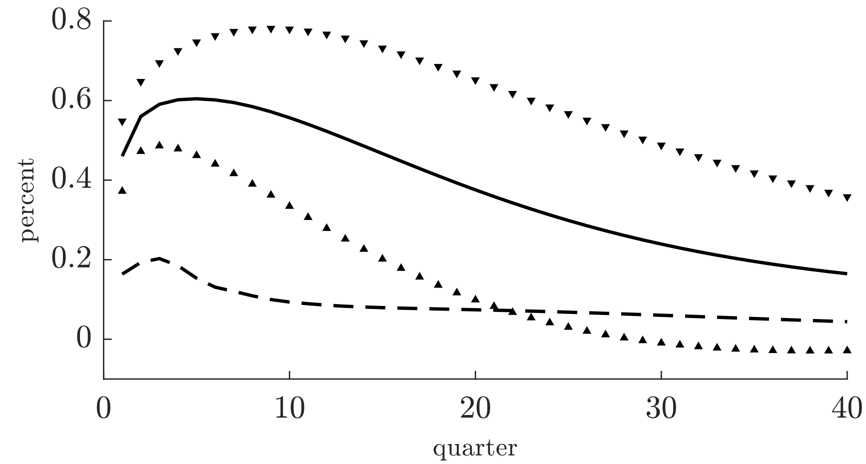
(c) Real-rate endpoint—Shocks (T-V loadings)



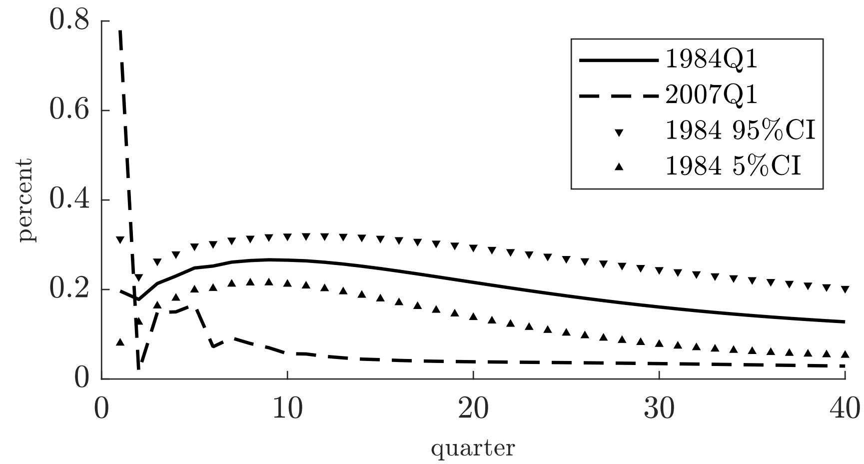
(d) Real-rate endpoint—Shocks (constant loadings)

Figure 5: Nominal Shocks Impulse Response Functions – Macro Variables

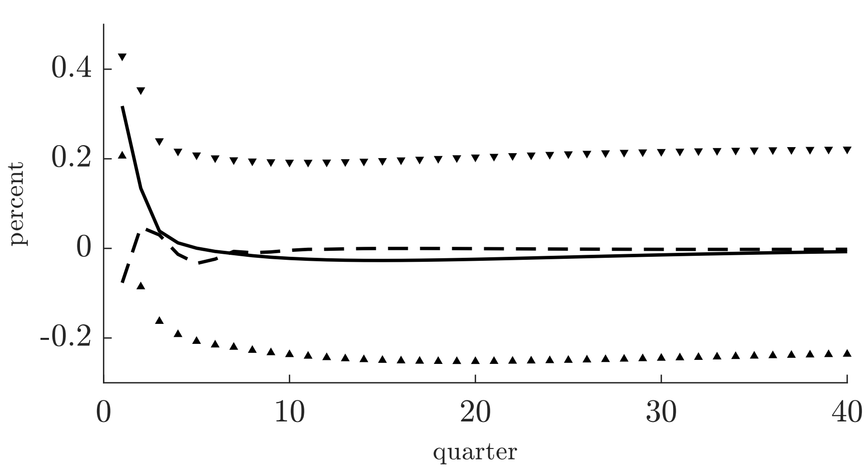
Impulse response functions with respect to the nominal shocks at horizons from 1 to 40 quarters from the model with time-varying loadings (the baseline model) on two different dates in our sample. Panel (a) shows the impulse responses function of the short rate i_t. Panel (b) shows the responses of the inflation rate π_t. Panel (c) shows the response of the output growth rate g_t. Panel (d) shows the response of the output gap.



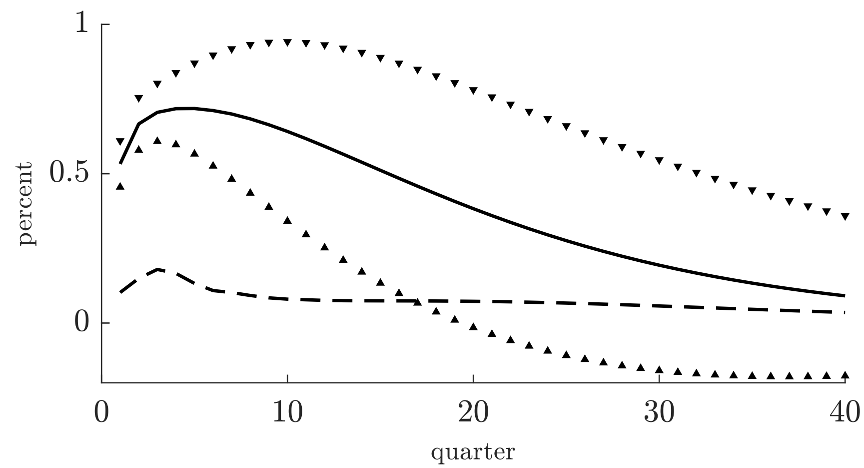
(a) Short rate



(b) Inflation



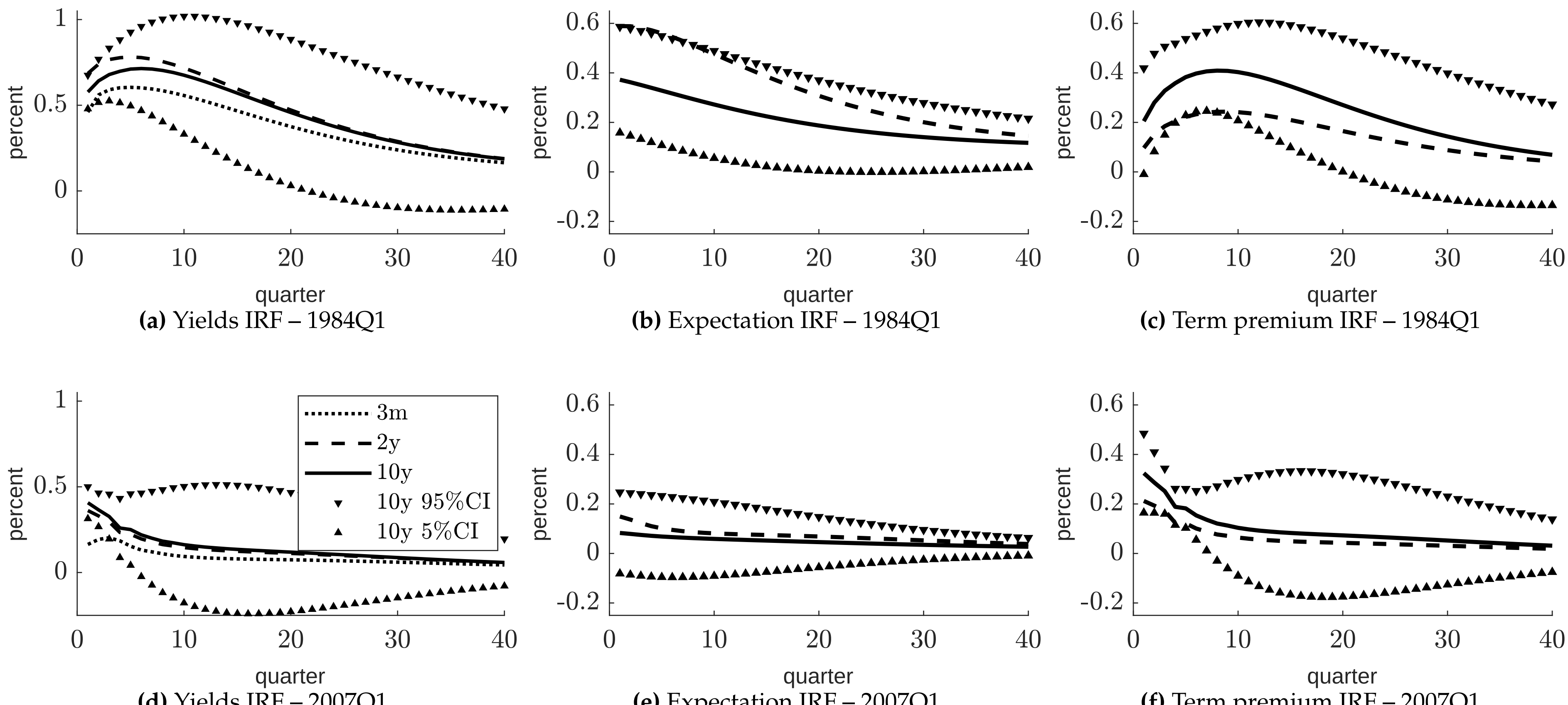
(c) Output growth



(d) Output gap

Figure 6: Nominal Shocks Impulse Response Functions – Bond Yields

Impulse response functions with respect to nominal shocks at horizons from 1 to 40 quarters at two different dates in our sample. Panels (a)-(e) show the responses of the yields, expectation and term premium components for bonds with 3 month (the short rate), 2 years and 10 years to maturity on 1984Q1. Panels (b)-(f) show the same responses but for 2007Q1. All units are in percentage points.



(a) Yields IRF – 1984Q1

(b) Expectation IRF – 1984Q1

(c) Term premium IRF – 1984Q1

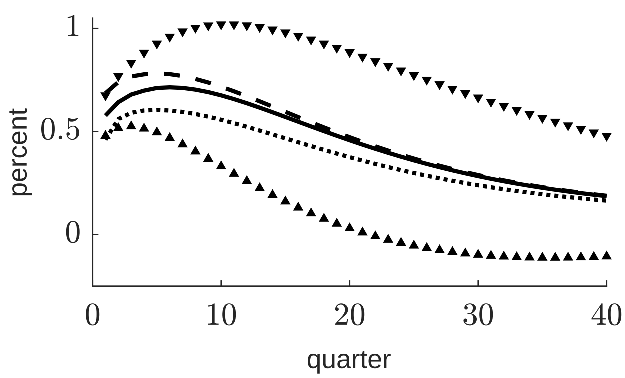
(d) Yields IRF – 2007Q1

(e) Expectation IRF – 2007Q1

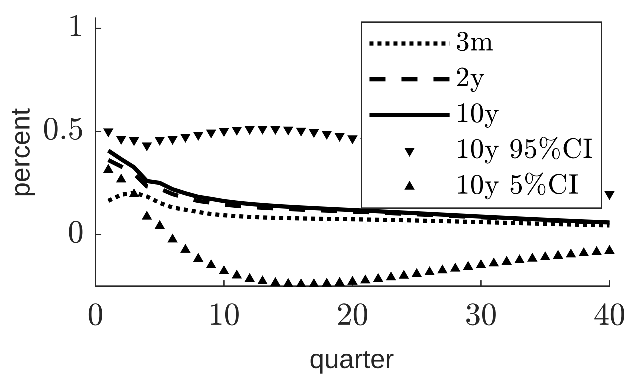
(f) Term premium IRF – 2007Q1

Figure 7: Nominal Shocks Impulse Response Functions – Bond Yields

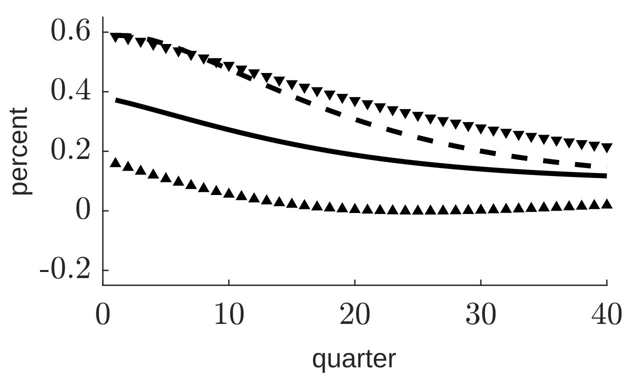
Impulse response functions with respect to nominal shocks at horizons from 1 to 40 quarters at two different dates in our sample. Panels (a)-(b) show the responses of the yields for bonds with 3 month (the short rate), 2 years and 10 years to maturity on 1984Q1 and 2007Q1, respectively. Panels (c)-(d) show the responses of the expectation components. Panels (e)-(f) show the responses of the term premium. All units are in percentage points.



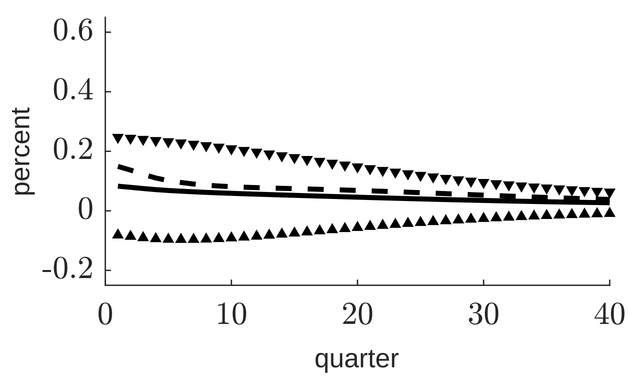
(a) Yields IRF – 1984Q1



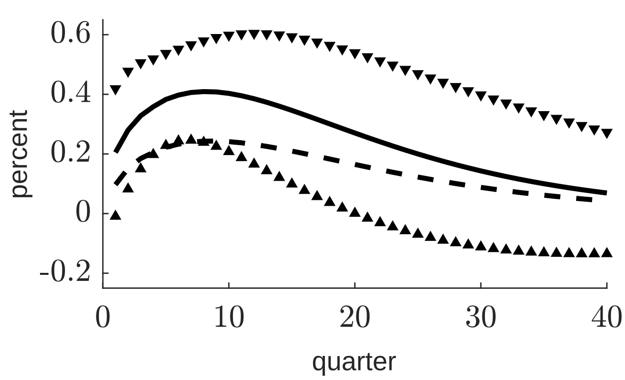
(b) Yields IRF – 2007Q1



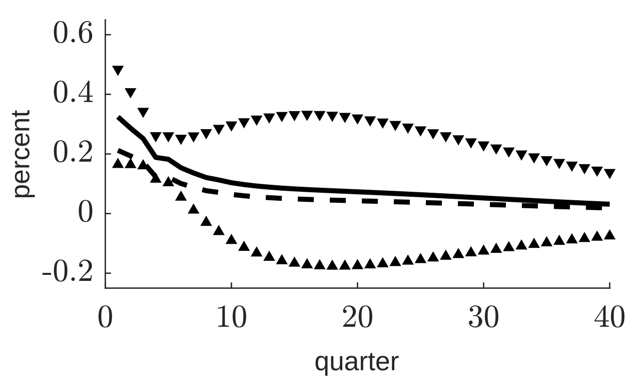
(c) Expectation IRF – 1984Q1



(d) Expectation IRF – 2007Q1



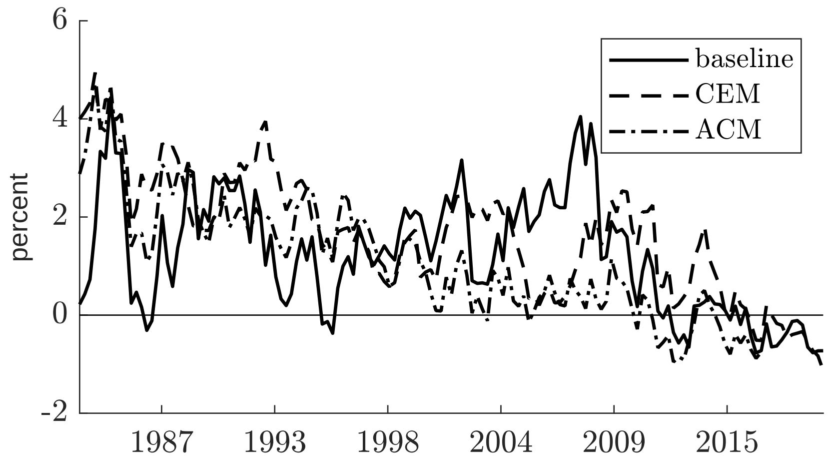
(e) Term premium IRF – 1984Q1



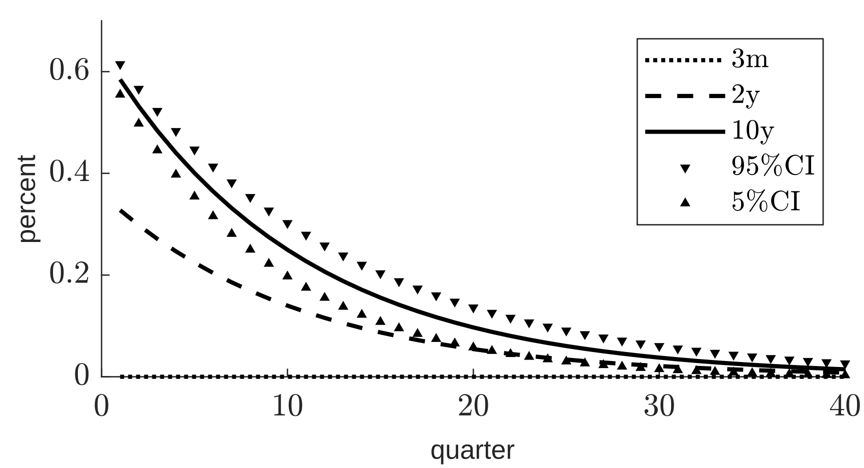
(f) Term premium IRF – 2007Q1

Figure 8: Term Premium

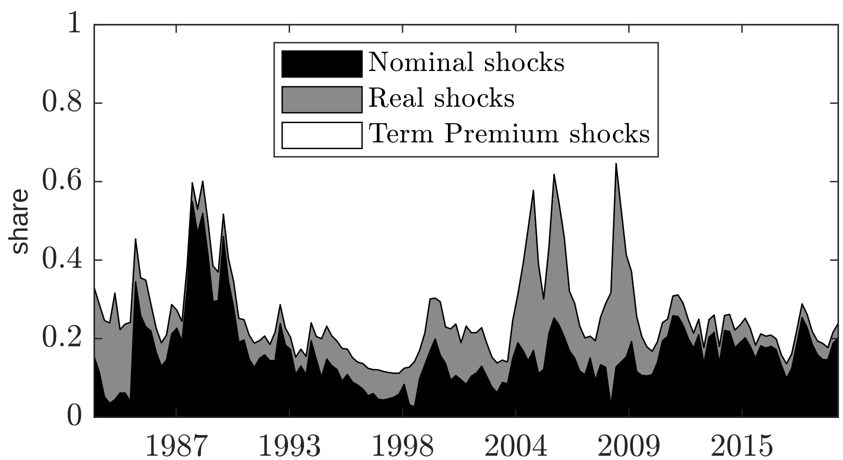
Panel (a) shows term premium estimates in percentages from the baseline model, from Adrian, Crump and Moench (2013) and from Crump, Eusepi and Moench (2016), labeled ACM and CEM, respectively. Panel (b) shows the term premium impulse response function with respect to term premium shocks $\varepsilon_{f,t+1}$ from our baseline specification. Panel (c) shows the term premium variance decomposition at the 4-quarter horizon, over time. Panel (d) shows the variance decomposition in the first quarter of 2007 at horizons between 1 and 40 quarters.



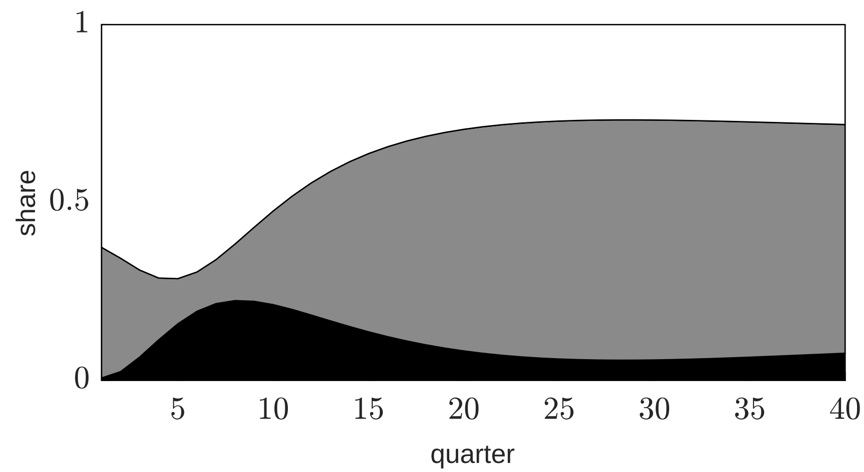
(a) 10-year term premium



(b) Term premium IRF



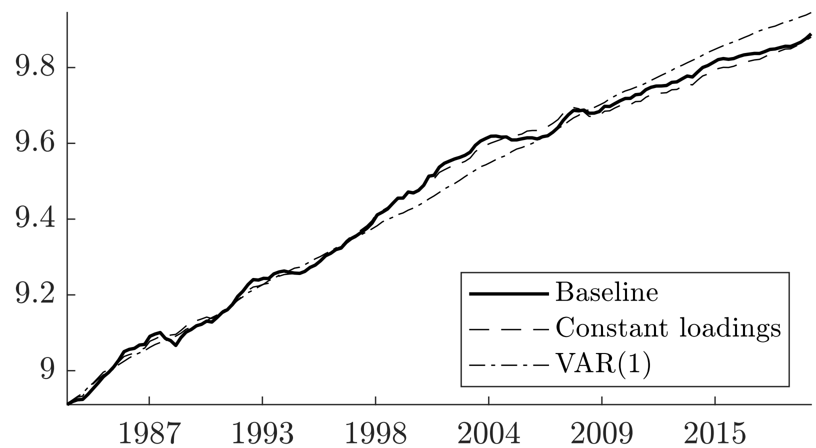
(c) Variance decomposition over time (one-year horizon)



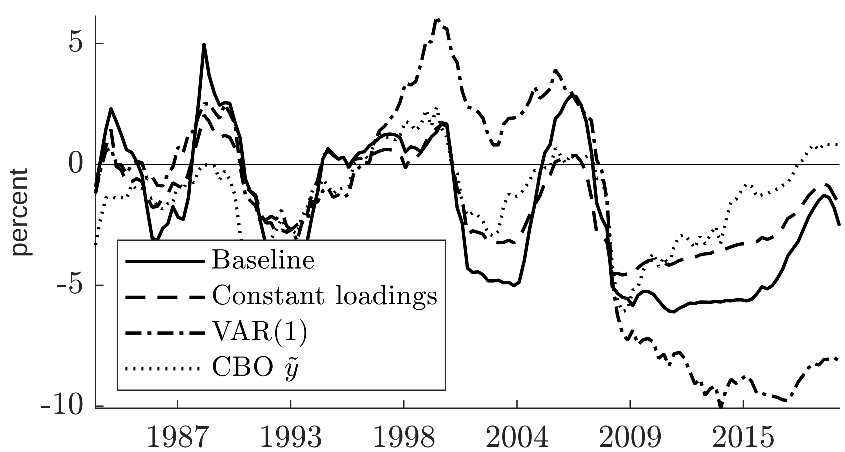
(d) Variance decomposition across horizons (2007Q1)

Figure 9: Estimates from a Model with VAR(1) Dynamics

Results in a model with VAR(1) cyclical dynamics and constant loadings. Panel (a) compares the potential output y_t^* estimates (units are in log-dollar) from the baseline model, the model with constant loadings and the VAR(1) model with constant loadings. Panel (b) compares the output gap $\tilde{y}$ estimates from the same model as well as the CBO estimates (units are in percentage points).



(a) Potential output



(b) Output gap

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